Analysis of inertial amplification mechanism with smart spring-damper for attenuation of beam vibrations

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Abstract. In this paper an inertial amplification mechanism with an embedded smart spring-damper device for attenuation of longitudinal vibrations in continuous structures is analyzed. The complex systems are the extension of the already investigated inertial mechanism, here additionally equipped with the vacuum controlled spring-damper device which shows features of smart materials. This allows the semi-active control to affect different frequency vibration ranges in the real time. The fea.tures of the basic inertia amplification mechanism are preserved as a possibility to generate two neighbouring anti-resonance frequencies between resonance peaks in the low frequency range.

Keywords: inertial amplification mechanism, vacuum packed particles, damping of vibration, smart structures.

1 Introduction

This work is an attempt to combine the local resonance phenomena and a spring-damper device with vacuum packaged granular materials to mitigate rod longitudinal vibrations. The semi-active system allows to control the affected vibration frequencies in the real time by changing the level of underpressure in the spring-damper device, what leads to the different values of the stiffness and damping coeffcient of the device.

The concept of inertial amplification was presented by Flannely in 1967, in a patented device for attenuating vibrations called a ”Dynamic Antiresonant Vibration Isolator” DAVI) [1]. The device is similar to a single degree of freedom (sdof) mass-spring system [2], however contains an additional mass, which is embedded between the main mass and the ground via a lever. This leaver amplifies an inertial force generated by isolation mass, and whole system works as a mechanical filter. The main mass is effectively isolated form ground vibrations where a transmissibility drops to zero for the particular frequency. The transmissibility of the standard spring-mass system for no frequency is reduced to zero. It rather approaches to zero for infinite frequencies. Furthermore, the standard spring-mass system is tuned to attenuate vibrations in a particular frequency range by the spring stiffness and its mass, where the DAVI system is tuned by the isolation mass and lever

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ratio, which are more practical tuning parameters. We can increase the lever ratio to increase the inertial force from the mass. This means that using an additional small mass, amplified ”effective” mass can be fairly large whereby an antiresonance frequency is obtained in the low frequency range.

The concept of inertial amplification was further developed by researches, however received less attention in the literature than local resonance phenomena. In 2002, Smith developed the inerter, which working principle is similar to Flannelly’s system. The inerter is based on a generated inertial force proportional to the relative acceleration between two attachment points for vibration isolation purposes. The invention has been utilized in practical applications as a suspensions systems of Formula One cars [3].

In the next couple of years Yimaz and collaborators have presented series of papers regarding an inertial amplification phenomena focusing on periodically repeated inertial mechanisms. As an example, they investigated a periodical array of system similar to Fllanylly’s DAVI to generate multiple anti-resonances and wide attenuated bands [4]. The next works concern lattice structures, where periodically repeated inertial mechanisms were a spine of vibration isolating structures [5, 6, 7].

The first attempts of utilizing the inertial amplification to attenuate vibrations in continuous structures have been presented by Frandsen and collaborators [8]. The inertial mechanism (similar to the single mechanism shown in [6]) is used to mitigate propagation of longitudinal waves through continuous 1D rod. The obtained results have shown that the system isolates vibrations at the low frequency range by generating wide and deep band gaps without applying a large isolation mass.

The system presented by Frandsen and collaborators [8] concerns a passive device for vibration attenuation. Here, a smart spring-damper is embedded to Frandsen’s mechanism to create a semi-active system for mitigation of longitudinal vibrations in 1D rod. This allows a control of affected frequencies by the hybrid system in the real time. Additionally, due to damping properties of the smart spring-damper device the resonance amplitude is effectively decreased. In this paper the concept of a vibration damping prototype with variable dissipation properties, based on a core made of Vacuum Packed Particles (VPP) is taken into consideration [9], [10], or [11]. These structures are constructed of granular materials (loose grains) placed in the hermetic plastomer envelope in which a partial vacuum is generated. The difference between the atmospheric pressure surrounding the granular core and the underpressure inside the system provides friction forces between the individual grains.

These forces are all the greater the lower the internal pressure exists in the core. Bearing in mind the obvious limitations, limited by atmospheric pressure and vacuum, the vacuum parameter allows the free and smooth change of the global physical properties of VPP. In the previous papers of the authors the particular attention was paid to the analysis of the experimental results obtained for the family of granulated cores filled with granular grains. Cyclic loading of samples applying a kinematic excitation rule was considered. The studies were conducted for three different excitation frequencies and a stepwise alternating, full range of possible underpressures. Additionally, attention was paid to the basic problems encountered during the research and ways to eliminate them in the proposed solution of the damper prototype. The most important, from the point of view of potential engineering applications, are the characteristics of the cores made of special granular structures. Typical examples are depicted in Fig. 1. The graph illustrates the effect of the underpressure on the dissipative capacities of VPP.

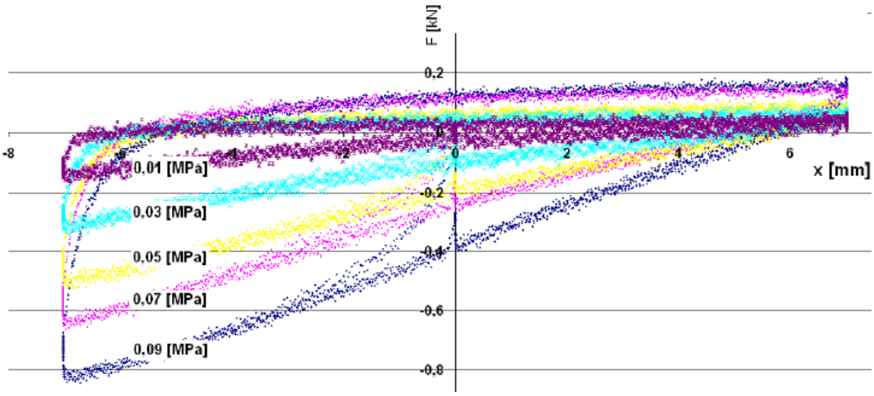

Fig. 1. Typical characteristics for VPP damper investigated under various values of underpressure

Based on the data illustrated in Fig. 1, we note that the influence of the vacuum parameter is unquestionable. Changing the vacuum parameter from $p_1 = 0.01$ MPa to $p_2 = 0.09$ MPa results in an increase in the recorded maximum force by several hundred percent. VPP proved to be an effective semi-active material for reducing vibration of beam elements [12], [13]. In the papers the authors used a granular sleeve in which the damped object was placed. The variations in internal vacuum had a significant effect on the dissipative properties of the system [14]. Compared to commercially available MR dampers and shock absorbers with variable dissipation properties, the proposed granular damper has many advantages. The main advantage is the economic aspect. In addition, the granular device appears to be less sensitive to temperature variations compared to the popular MR damper [15]. Another advantage of the proposed solution is its reduced sensitivity to pollution. In addition, in the case of granular conglomerates, the problem of tightness is automatically eliminated. The basic limitation of granular damper is its nonlinear characteristics. However, this obstacle can easily be eliminated by changing the design of the device.

2 Modelling

The attached hybrid mechanism to the elastic 1D rod by simple supports is presented in Fig. 2. Generally, the hybrid system can be split into to two separate devices i.e. the IA mechanism and a smart spring-damper. The first one consists of the bars denoted by l, which are connected by a moment free connection at the top and their inclination is controlled by an angle Θ_l. Furthermore, it is assumed that the bars are ideally rigid and massless to avoid their deformation. The isolating mass ma generates the inertial force. As it can be seen in Fig. 2, the spring-damper is attached between bars of the IA and is described by following parameters: a stiffness coefficient $k(p)$ and a damping coefficient $c(p)$, where p denotes an underpressure. All elements of the hybrid system are connected together by moment free connections. This provides that no moment is transferred through the system and allows for the elastic rod to vibrate freely. The elastic rod with Young modulus E, mass density ρ, length L has rectangular cross section with thickness t_b and width w_b.

This is an initial simplified mechanical model of the hybrid system, where the masses of the smart damper, rigid bars and bearings are omitted. Moreover, the physical construction of the spring-damper device is not investigated in this article. The attention is given on the impact of smart device in case of the control range of the affected vibration frequencies and whether it is whatsoever possible.

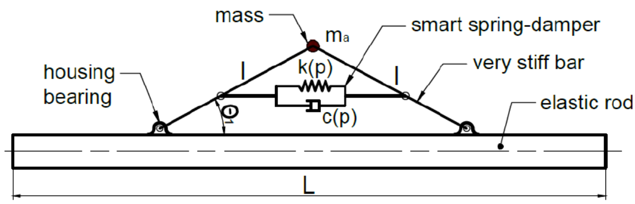

Fig. 2. The elastic rod with attached hybrid mechanism

3 Analysis

The effect of the hybrid system on the rod vibrations is discussed in this section. The parameters of the system are investigated and their impact on the hybrid mechanism effectiveness i.e., a control scope of the affected vibration frequencies is revealed.

The FE-model of the hybrid system is created in commercial FE-software Ansys. The host rod as well as bars are modeled by three-dimensional beam elements, which are based on Timoshenko beam theory. The bars are connected together and with the host beam by revolution joints to have moment free connections. The additional mass of the hybrid system is applied as a point mass at the top nodes of the attached mechanism. A spring-damper element is used to model the smart damping device, which is massless. As it can be seen in Fig. 3 the hybrid system is attached to host beam at the top and underneath. This ensures that vertical force components, which are generated by both mechanisms cancel out each other. Hence, only pure horizontal forces are transmitted to the host beam.

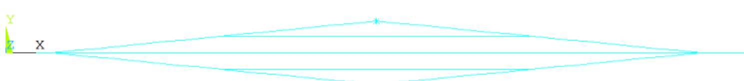

Fig. 3. FEM model of the system in Ansys environment

The system is excited to the vibrations by a periodical force with an amplitude F_0 = 100 [N], which is applied at the first node of the rod in the horizontal direction. The response of the system i.e., an amplitude of vibrations is examined at the second beam end. The host beam always has the same material parameters given in Tab. 1. The rigidity of the bars is obtained by setting their Young modulus $E_b = 1 \times 10^{20}$ [Pa]. Here, we only focus on the simplified system having rigid bars, of course for a physical system bars parameters will have an impact on mechanism performance in case of the vibration attenuation. This phenomena was investigated by Yilmaz and Hulbert [6], where authors show that their IA is the most effective for anti-resonance frequencies, which are looked for in the frequency range below an IA resonance frequency.

The all investigated parameters of the system are illustrated in Fig. 4.

Table 1. Beam mechanical properties.

L [m]	w_b [m]	t_b [m]	E [Pa]	ρ [kg/m^3]
0.8	*0.05*	*0.005*	2.1×10^{11}	*7850*

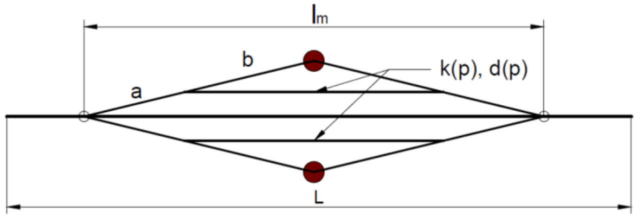

Fig. 4. System with analysed parameters

First, the effect of attaching the hybrid system to the rod is presented for mechanism parameters given in Tab. 2, where M_b is a mass of the rod and a damping ratio is determined by $\xi = c(p)/2\sqrt{m_a k(p)}$. Fig. 5 shows three FRF's: for a homogeneous rod, the rod with pure IA and the rod with the hybrid system. The excitation frequency is normalized by the first resonance frequency of the homogeneous rod and is denoted by $\overline{\omega}$.

Table 2. Mechanism parameters – case No. 1

$\boldsymbol{\theta_1}$ **[rad]**	$\boldsymbol{m_a/M_b}$	$\boldsymbol{l_m/L}$	$\boldsymbol{k}$ **[m]**	$\boldsymbol{\xi}$	$\boldsymbol{\overline{h}}$
$\pi/27.6$	*0.09*	*0.875*	4×10^8	*0.28*	*0.5*

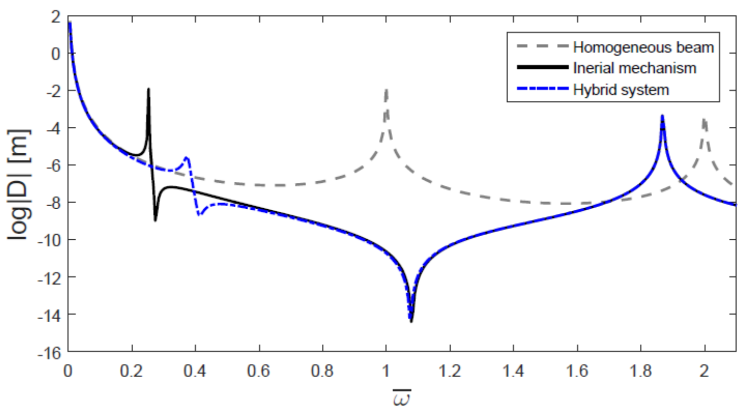

Fig. 5. Comparison of the FRF's for different systems

The black FRF in Fig. 5 shows the vibration attenuation properties of the inertial amplification mechanism i.e., it generates anti-resonance dips in the low frequency range. Furthermore, two neighboring dips are observed between two resonance peaks. This is characteristic behavior of the IA system and is described accurately in [8]. Here, we focus on the smart damper which is added to the IA mechanism construction. As it is seen in Fig. 5, for the hybrid system, the first resonance peak and anti-resonance dip are moved to the right (in dir. of the higher frequencies) in comparison to the FRF of the pure IA mechanism. This is caused by the additional spring element, which increases the stiffness of the whole system. These peak and dip for the hybrid system are no longer sharp but they are rounded and attenuated to the lower level. It means that the smart damper suppresses not only resonance vibrations but also the anti-resonance dips, which the amplitude does not approach zero any more.

What is worth emphasizing, only the first peak and the dip are affected by the smart-damper and the second order peak and dip still have the same trend as for the IA. It can be explained by a vibration form for the particular mode shape. For the first mode of vibration the attachment points of the damper device move out-of phase in horizontal direction, thus their net relative motion has maximal value and device in the most effective. For the second mode the attachments points move in-phase, thus relative motion seen by the damper device is insignificant and device does not work.

Based on Fig. 5 we can see that the smart damper embedded into IA works and can be used as an additional element to control the range of affected vibration frequencies by the system. Furthermore, it attenuates resonance peaks what is its advantage, does not seen for the pure IA mechanism. In the further sections, the particular parameters of the smart damper are analyzed and their impact on the FRF.

3.1 Hybrid system stiffness variation

The first analysed parameter is the stiffness of the hybrid system. The damping is set to zero and other parameters are given in Tab. 2. Fig. 6 shows FRF's for the pure IA as well as for the hybrid systems with different values of the stiffness coefficient.

As it can be seen in Fig. 6, increasing stiffness of the spring we move the first resonance peak and anti-resonance dip to higher frequencies. This proves that the stiffness coefficient of the spring-damper device can be a tuning parameter, and the system works with the expectations.

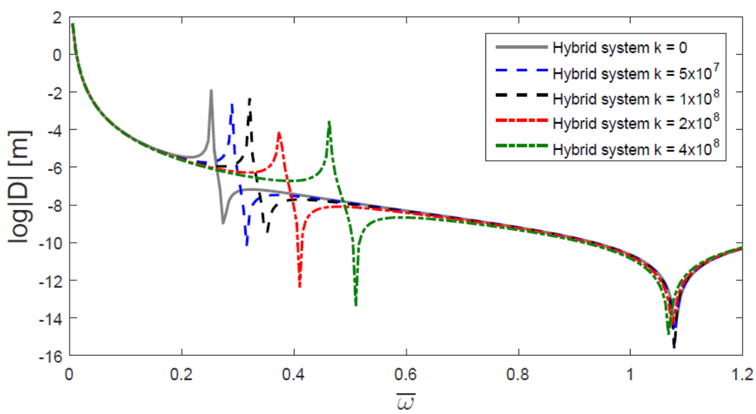

Fig. 6. Stiffness variation of the spring-damper device

3.2 Hybrid system damping variation

The damping coefficient is the next investigated parameter of the hybrid system. The FRF's are plotted in Fig. 7 for the all other parameters presented in Tab. 2.

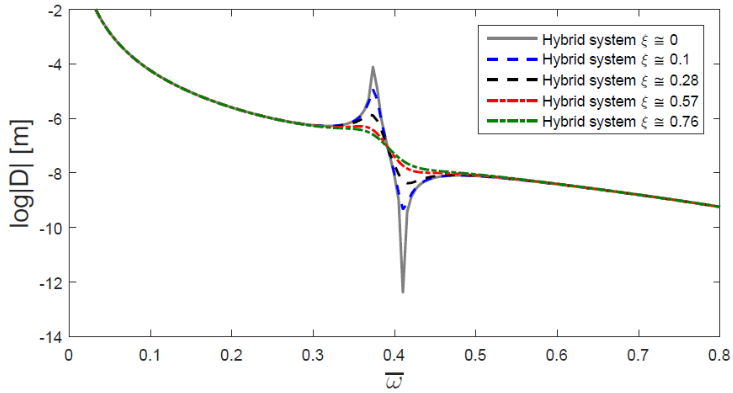

Fig. 7. Damping variation of the spring-damper device

Fig. 7 shows that the damper decrease, both resonance and anti-resonance amplitude of vibrations. The impact of the damper is the same as for Flannely's DAVI, where damping element has affected both resonance peak and anti-resonance dip [1]. Even though the damper decrease the level of the attenuation for the anti-resonance dip, still its effect is considered as a positive because the resonance peak which occur as a first along the FRF does not rise to the infinite value.

3.2 Effect of smart damper location

The last considered parameter is the height of the attachment points of the smart damper device. Fig. 8 presents FRF's for the system with a varying parameter $h = a/(a + b)$, where variables a and b are illustrated in Fig. 4. It is expected that the location of the attachment point along bar l can have significant impact on the affected vibration frequencies. The distance b works as a lever arm for forces generated by the spring-damper device and amplifies them for the increasing distance b.

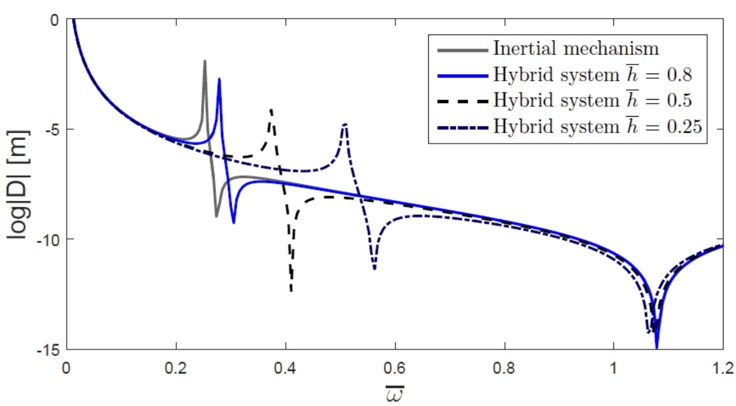

Fig. 8. Effect of the attachment points of the spring-damper device

As it is seen in Fig. 8, the obtained results are in line with above prediction. Decreasing parameter h, we can considerably increase the frequency of the first resonance peak and anti-resonance dip. Thus, in order to observe the maximal effect of the spring-mass damper, it has to be attached as close to the rod as possible. Then its stiffness can be lower, what is an advantage in the case of longitudinal vibrations of the continuous structures. Where a large stiffness is necessary to see the effect of the external device as e.g. the standard local resonator [2].

Conclusion

The hybrid system for attenuation of the longitudinal vibrations in the continuous rod has been analyzed numerically. It was shown that applying the smart spring-damper to the inertial mechanism we can control the frequency of the first resonance peak and anti-resonance dip. Furthermore the hybrid system suppresses the amplitude of the resonance and anti-resonance vibrations. Of course the all 'trademarks' of the pure IA mechanism [8] are preserved i.e., the possibility of generating two neighboring dips between resonance peaks or multiple anti-resonance dips in the FRF. Basically, the hybrid system extends the possibilities of the pure IA mechanism, develops it from the passive to the semi-active system. The spring-damper device embedded in the IA mechanism construction allows the semi-active control of the affected frequencies in the real time by the hybrid system. A physical realization of the device assumes using the under-pressure as a control factor to change stiffness and damping coefficients of the device. The device will be based on the vacuum packaged particles. The major challenge with the construction of the damper device is its stiffness, which has to be high enough to see the effect of the hybrid system in case of the control affected frequencies for longitudinal vibration. As it was shown in Fig. 8, the force generated by damper device is amplified by the construction of the system, however still the large stiffness will be necessary.

Future work assumes to realize physical model of the system and conduct experiments e.g. the modal analysis. Furthermore, the hybrid system can be applied to the continuous beam to attenuate bending vibrations and control the affected vibration frequencies.

References

1. G. Flannelly, *Dynamic antiresonant vibration isolator*. U.S. Patent 989, 958 (1967)
2. H. Frahm, *Device for damping vibration of bodies*. U.S. Patent No. 989958 (1911)
3. M. Chen, C. Papageorgiou, F. Scheibe, F.-C.Wang, M. Smith, *The missing mechanical circuit element*. IEEE Circuits and Systems Magazine **9**, 10-26 (2009)
4. C. Yilmaz, N. Kikuchi, *Analysis and design of passive low-pass filter-type vibration isolators considering stiffness and mass limitations*. Journal of Sound and Vibration **293**, 171-195 (2006)
5. C. Yilmaz, G. Hulbert, N. Kikuchi, *Photonic band gaps induced by inertial amplification in periodic media.* Physical Review B **76**, 054309 (2007)
6. C. Yilmaz, G. Hulbert, *Theory of phononic gaps induced by inertial amplification in finite structures*. Physics Letters A **374,** 3576–3584 (2010)
7. G. Acar, C. Yilmaz, *Experimental and numerical evidence for the existence of wide and deep photonic gaps induced by inertial amplification in two-dimensional solid structures*. Journal of Sound and Vibration **332**, 6389–6404 (2013)
8. N. M. M. Frandsen, O. R. Bilal, J. S. Jensen, M. I. Hussein, *Amplification of continuous structures: Large band gaps from small masses*. Applied Physics **119**, 124902 (2016)
9. R. Zalewski, P. Chodkiewicz, *Semi-active linear vacuum packed particles damper.* Journal of Theoretical and Applied Mechanics **54** (1), 311–316 (2016)

10. R. Zalewski, M. Shillor, P. Chodkiewicz, *Vibrations of a mass-spring system using a granular-material damper*. Applied Mathematical Modelling **40**, 8033–8047 (2016)
11. M. Makowski, R. Zalewski, *Vibration analysis for vehicle with vacuum packed particles suspension*. Journal of Theoretical and Applied Mechanics **53** (1), 109-117 (2015)
12. R. Zalewski, T. Szmidt, *Application of special granular structures for semi-active damping of lateral beam vibrations*. Engineering Structures **65**, 13–20 (2014)
13. T. Szmidt, R. Zalewski, *Inertially excited beam vibrations damped by vacuum packed particles*. Smart Mater. Struct. **23** (9), (2014)
14. J. Bajkowski, R. Zalewski, *Transient response analysis of a steel beam with vacuum packed particles.* Mechanics Research Communications **60**, 1–6 (2014)
15. R. Zalewski, J. Nachman, M. Shillor, J. Bajkowski, *Dynamic model for a magnetorheological damper*. Applied Mathematical Modelling **38**, 2366-2376 (2014)